AIAA JOURNAL
Vol. 43, No. 11, November 2005

Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Impedance Modeling Technique for a Fluid-Loaded Structure

C. C. Cheng* and P. W. Wang†
National Chung Cheng University,
Chia-Yi 621, Taiwan, Republic of China

I. Introduction

FOR a fluid-loaded structure, how to calculate the coupled motions between the elastic structure and the associated medium that surrounds it has attracted many researchers for many years.[1–8] Different methods that include analytical approaches (e.g., the complex integral transform, asymptotic technique), and the numerical techniques [e.g. the finite element method (FEM), the boundary element method (BEM)], etc., are available, and each has its own advantages. Most of them derive the equation of motion of the fluid-loaded structure first and then find the associated vibro-acoustic response. However, great inherent difficulties in the interaction between the fluid and the structure are encountered when the structure either has an irregular shape or is subject to boundary conditions that are so complicated even the numerical technique, for example, FEM or BEM, has trouble in modeling them.

This Note proposes an impedance technique[9] to obtain the dynamic response of a fluid-loaded structure. The structural surface is divided into multiple segments, and then the impedance of each structural segment and the fluid are investigated individually. By applying the conditions of force equilibrium and response compatibility between the structure and the fluid, the impedance of the fluid-loaded structure can be expressed as the impedance couplings between the fluid and the structure. For systems where either the equations of motion or the associated boundary conditions are difficult to be modeled, the impedances of the structure and fluid can be obtained from experimental measurements. The impedance of a mechanical component expressed using the frequency response function (FRF) links the associated analytical model to practical measurements, which has been demonstrated by traditional modal testing. Therefore, it provides an alternative to obtaining the vibroacoustic response of a fluid-loaded structure.

Nevertheless, for a fluid-loaded structure, how to model the fluid-loading effect and then couple it with the impedance of the structure in vacuo is still an issue that must be answered first. It is well known that the fluid-loading effect on a structure can be modeled using Fourier transform as an acoustic wave impedance in the wave number domain. However, the impedance of the structure in vacuo is expressed using the frequency response function and thus it is required to express the fluid loading in terms of impedance in the same FRF model as well. In this Note, a methodology of expressing the fluid loading as a radiation impedance on the structure is developed. A formulation that assembles the radiation impedance and the structural impedance is introduced in order to demonstrate how a fluid-loaded structural impedance model is constructed by using the impedance of structure in vacuo and the radiation impedance.

II. Mechanical Mobility of Structures

Although the definition of mechanical mobility is well known, for the purpose of completeness, the concept of structural mobility will be introduced here using a simple beam as an example.[10,11] Consider a beam of length L, thickness b, width w, and density ρ_0, whose surface is divided into N segments as shown in Fig. 1. When a harmonic force with a frequency ω acts on the midpoint of the nth segment, the equation of motion for the beam is expressed as

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = F_v\delta(x - x_n)e^{i\omega t} \tag{1}$$

where $\delta(x)$ is the Dirac delta function, w the beam displacement, EI the bending rigidity, ρA the mass per unit length, F_v the concentrated force, and x_n the coordinate corresponding to the midpoint of the nth segment. Based on the information of excitation at x_n and the response at x_m, the mobility of the beam $m_{m,n}$ is defined as

$$m_{m,n} = \frac{V_v(x_m, t)}{F_v e^{i\omega t}} = \sum_{k=1}^{\infty}\frac{i\omega\phi_k(x_n)\phi_k(x_m)}{\left(\omega_k^2 - \omega^2\right)} \tag{2}$$

where $V_v(x_m, t)$ is the transverse velocity of the mth beam segment, ω_k is the kth natural frequency of the beam, and ϕ_k is the corresponding mass normalized shape function that satisfies the associated boundary conditions. The first subscript of m indicates the response segment, and the second denotes the excitation segment, respectively. Based on the concept of frequency response function,[12] the force-velocity relation of the beam can be rewritten as

$$\boldsymbol{V}_v = \boldsymbol{MF} \tag{3}$$

where $\boldsymbol{V}_v - \{V_{v1}, V_{v2}, V_{v3}, \ldots, V_{vN}\}^T$ is the beam transverse velocity represented discretely by the velocity of each beam segment, $\boldsymbol{F} = \{F_1, F_2, F_3, \ldots, F_N\}$ is the external force acting on each beam segment, for example, $\boldsymbol{F} = 0, 0, 0, \ldots, F_n = F_v, \ldots, 0\}$ in this example, and $\boldsymbol{M}$ is the structural mobility expressed as

$$\boldsymbol{M} = \begin{bmatrix} m_{1,1} & m_{1,2} & .. & m_{1,N} \\ m_{2,1} & m_{2,2} & .. & .. \\ .. & .. & m_{m,n} & .. \\ m_{N,1} & .. & .. & m_{N,N} \end{bmatrix}_{N\times N} \tag{4}$$

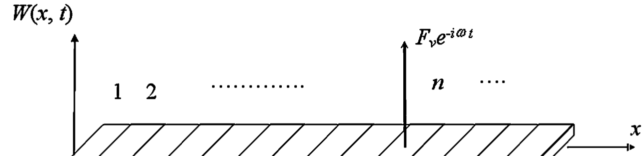

Fig. 1 Beam subjected to a concentrated harmonic force at the *n*th segment.

Received 11 April 2005; revision received 16 June 2005; accepted for publication 16 June 2005.

*Professor, Department of Mechanical Engineering, 160, San-Hsing, Ming-Hsiung; imeccc@ ccu.edu.tw.
†Graduate Research Assistant, Department of Mechanical Engineering, 160, San-Hsing, Ming-Hsiung.

Equation (4) provides an alternative to finding the structural response, provided that the mobility $\boldsymbol{M}$ is known a priori. When the structure is either in an irregular shape or has complicated boundary conditions, $\boldsymbol{M}$ is much more easily determined using an experimental technique, for example, by impact testing,[12] rather than an analytical method. However, the number of segments used to divide a structure, the so-called spatial resolution, depends on the excitation frequency, which can range from 10 to several hundred. It implies that constructing the mobilty matrix $\boldsymbol{M}$ is a time-comsuming process. Fortunately, the number of measurements can be reduced dramatically due to reciprocity.

III. Impedance of Fluid

Consider that a baffled, finite fluid-loaded beam vibrates with a transverse velocity $V(x)$; the acoustic pressure at the beam surface can be expressed as[8]

$$P_a(x,0) = \frac{\rho_0\omega}{2\pi}\int_{-\infty}^{\infty}\frac{\bar{V}(\alpha)}{\sqrt{k_0^2-\alpha^2}}e^{j\alpha x}\,\mathrm{d}\alpha \tag{5}$$

where k_0 is the acoustic wave number, $\bar{V}(\alpha)$ is the Fourier transform of the beam velocity $V(x)$:

$$\bar{V}(\alpha) = \int_{-\infty}^{\infty} V(x)e^{-j\alpha x}\,\mathrm{d}x \tag{6}$$

and the branch of the radical is defined by

$$\sqrt{k_0^2-\alpha^2} = \begin{cases}\sqrt{k_0^2-\alpha^2}, & k_0 \ge \alpha\\ -j\sqrt{\alpha^2-k_0^2}, & \alpha > k_0\end{cases} \tag{7}$$

Assume that the beam is divided into N segments, and each vibrating beam segment can be regarded as a sound source. The transverse velocity of the mth beam segment vibrating with a unit amplitude is expressed as

$$v_m(x) = [H(x-x_{1m}) - H(x-x_{2m})]e^{j\omega t} \tag{8}$$

where H is the Heaviside unit function and x_{1m} and x_{2m} are the coordinates corresponding to the left and the right ends of the mth beam segment. The Fourier transform of Eq. (8) is

$$\bar{V}_m(\alpha) = \frac{e^{-j\alpha x_{2m}} - e^{-j\alpha x_{1m}}}{-j\alpha} \tag{9}$$

Substituting Eq. (9) into Eq. (5) yields the acoustic pressure acting on the beam surface:

$$P_a(x,0) = \frac{\rho_0\omega}{2\pi}\int_{-\infty}^{\infty}\frac{\exp[-j\alpha(x-x_{2m})] - \exp[j\alpha(x-x_{1m})]}{-j\alpha\sqrt{k_0^2-\alpha^2}}\,\mathrm{d}\alpha \tag{10}$$

Equation (10) represents the acoustic pressure contributed solely by the mth beam segment that is vibrating with a unit amplitude at frequency ω. If the length of each beam segment is small compared with the acoustic wavelength of the fluid, the force acting on the nth beam segment as a result of the mth beam segment vibrating with transverse velocity v_{vm} can be represented equivalently as a concentrated force $F_{an,m}$ acting on the midpoint of the nth beam segment:

$$F_{an,m} = -\left[w\int_{x_{1n}}^{x_{2n}} P_{am}(x,0)\,\mathrm{d}x\right]v_{vm} \tag{11}$$

where w is the width of the structure, the negative sign represents the force induced by the acoustic pressure, x_{1n} and x_{2n} are the coordinates of the left and right ends of the nth segment, respectively. Equation (11) can be expressed using the impedance notation:

$$F_{an,m} = -Z_{an,m}v_{vm} \tag{12}$$

where

$$Z_{an,m} = w\int_{x_{1n}}^{x_{2n}} P_{am}(x,0)\,\mathrm{d}x$$

It is obvious that the value of $Z_{an,m}$ depends on the distance between the receiver for example, the nth beam segment, and the sound source, for example, the mth beam segment. Moreover, the reciprocity holds as the source and the receiver interchange:

$$Z_{am,n} = Z_{an,m} \tag{13}$$

Based on reciprocity, the source and receiver have a more general relation expressed as the following:

$$Z_{am,n} = Z_{a[\mathrm{abs}(m-n)+1],1} \tag{14}$$

For example, $Z_{a5,2}$ is equal to $Z_{a4,1}$ or $Z_{a1,4}$ because the distance between segment no. 5 and segment no. 2 is equal to that between segment no. 4 and segment no. 1. If all of the beam segments vibrate simultaneously, the forces caused by the associated acoustic pressure acting on each beam segment can be expressed as

$$\boldsymbol{F}_a = -\boldsymbol{Z}_a\boldsymbol{V}_v \tag{15}$$

where $\boldsymbol{F}_a = \{F_{a1}, F_{a2}, \ldots, F_{aN}\}^T$ is the transverse force acting on the midpoint of each segment, $\boldsymbol{V} = \{V_{v1}, V_{v2}, V_{v3}, \ldots, V_{vN}\}^T$ is the transverse velocity of each beam segment, and $\boldsymbol{Z}_a$ is the impedance of the fluid:

$$\boldsymbol{Z}_a = \begin{bmatrix} Z_{a1,1} & Z_{a1,2} & .. & Z_{a1,N}\\ Z_{a2,1} & Z_{a2,2} & .. & ..\\ .. & .. & Z_{am,n} & ..\\ Z_{aN,1} & .. & .. & Z_{aN,N}\end{bmatrix}_{N\times N} \tag{16}$$

Each row in Eq. (16) represents the sound pressure acting on a specific beam segment, whereas each column is the sound pressure radiated from a specific beam segment acting as a sound source. It is clear that Eq. (16) provides an alternative to determine the radiated acoustic pressure of a vibrating structure based on the impedance technique without deriving the equation of motion, provided that the velocity response of the beam is obtained antecedently. Notice that the sound pressure is acting on the beam as fluid loading and thus influences the beam velocity. How to couple the impedances between the structure and the fluid loading and thus have an accurate prediction of the vibroacoustic response of the fluid-loaded structure is described in the following section.

IV. Impedance Couplings Between Structure and Fluid

Consider the same fluid-loaded beam mentioned in the preceding section, but it is excited by the force $\boldsymbol{F}_e = \{F_{e1}\ \ F_{e2}\ \ ..\ \ F_{en}\ \ ..\ \ F_{eN}\}^T$, where F_{en} represents the concentrated force acting on the nth segment as shown in Fig. 2. The force acting on the structure includes the external force acting on each beam segment and the fluid loading as expressed in Eq. (15):

$$\boldsymbol{F} = \begin{Bmatrix}F_{e1}\\ F_{e2}\\ ..\\ F_{eN}\end{Bmatrix} - \begin{bmatrix} Z_{a11} & Z_{a12} & .. & Z_{a1N}\\ Z_{a21} & .. & .. & ..\\ .. & .. & .. & ..\\ Z_{aN1} & Z_{aN2} & .. & Z_{aNN}\end{bmatrix}\begin{Bmatrix}V_{v1}\\ V_{v2}\\ ..\\ V_{vN}\end{Bmatrix} \tag{17a}$$

Equation (17a) is rewritten by using matrices as

$$\boldsymbol{F} = \boldsymbol{F}_e - \boldsymbol{Z}_a\boldsymbol{V}_v \tag{17b}$$

where $\boldsymbol{F}_e$ causes the structure vibrating harmonically and $-\boldsymbol{Z}_a\boldsymbol{V}_v$ results from the fluid loading. Substitution of Eq. (17b) into Eq. (3) yields

$$(\boldsymbol{I} + \boldsymbol{M}\boldsymbol{Z}_a)\boldsymbol{V}_v = \boldsymbol{M}\boldsymbol{F}_e \tag{18}$$

where $\boldsymbol{I}$ is a $N\times N$ unit matrix. The term $\boldsymbol{I} + \boldsymbol{M}\boldsymbol{Z}_a$ represents the impedance coupling between the structure and the fluid, and $\boldsymbol{M}\boldsymbol{F}_e$

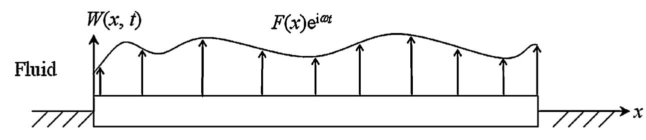

Fig. 2 Fluid-loaded beam subjected to distributed harmonic forces.

is the transverse velocity of the structure in vacuo caused by the external force. Equation (18) can be interpreted as a transformation of a structural response from a structure in vacuo, $\boldsymbol{MF}_e$ into that from a fluid-loaded structure $\boldsymbol{V}_v$. The velocity of the fluid-loaded structure is simply written in terms of the mobility of the structure in vacuo and the acoustic impedance of fluid:

$$\boldsymbol{V}_v = \boldsymbol{M}_{\text{fl}}\boldsymbol{F}_e \tag{19}$$

where $\boldsymbol{M}_{\text{fl}} = [(\boldsymbol{I} + \boldsymbol{MZ}_a)^{-1}\boldsymbol{M}]$ is the mobility of the fluid-loaded structure. The structure is assumed to be immersed in a homogeneous fluid at rest, that is, the influence of fluid motion on the structural response has been neglected. Therefore it is not every kind of fluid-structure interaction that can be formulated by using the proposed methodology. The proposed technique, nevertheless, provides an alternative on the modeling of fluid-structure interaction.

V. Numerical Validation

To illustrate the methodology and the formulation discussed in the preceding sections, an example is presented to validate the fluid loading calculated using the proposed methodology by comparing it to that obtained from a traditional method, for example, the wave-number transformation method.

Consider a fluid-loaded baffled beam with simply supported boundary conditions subject to a concentrated harmonic force acting at $x = 0.045$ m. The parameters of the beam and the fluid are listed in Table 1. Figure 3 shows the driving point mobility obtained using the proposed impedance method together with that from the transformation method. The dotted line denotes the mobility spectrum and is calculated with the beam being divided into 10 equal-length segments, whereas the dashed line represents the mobility obtained using 30 equal-length beam segments. Notice that the result using 10 equal-length segments agrees well with that from the transformation method at low frequency, and their discrepancies increase as the frequency increases. However, the discrepancy in the frequency spectra at high frequency decreases as the total segment number increases. It is not surprising that a smaller beam segment should be adopted in the proposed impedance method to accurately approximate the fluid loading for a high-frequency excitation.

Figures 4 and 5 show the acoustic impedances caused by the no. 1 and no. 50 beam segments at an excitation frequency of 200 and 3000 Hz, respectively, while the beam is divided into 100 equal-length segments. It is well known that the real part of the impedance has a damping effect on the beam, whereas the imaginary part has a mass effect. It is not surprising that the mass effect is localized near the source, the no. 1 and no. 50 beam segments in this example. On the contrary, both the no. 1 and no. 50 beam segments have similar damping effects on the other beam segments. The reason is simply that the acoustic wavelength is larger than the length of the beam when the excitation frequency is 3000 Hz.

Table 1 Parameters of beam and fluid

Parameter	Value
Beam	
Thickness h, m	2×10^{-3}
Length L, m	6×10^{-1}
Width b, m	5×10^{-2}
Young's modulus E, N/m^2	7×10^{10}
Density ρ, kg/m^3	2.7×10^3
Fluid	
Wave speed C, m/s	1.5×10^3
Density ρ_0, kg/m^3	1×10^3

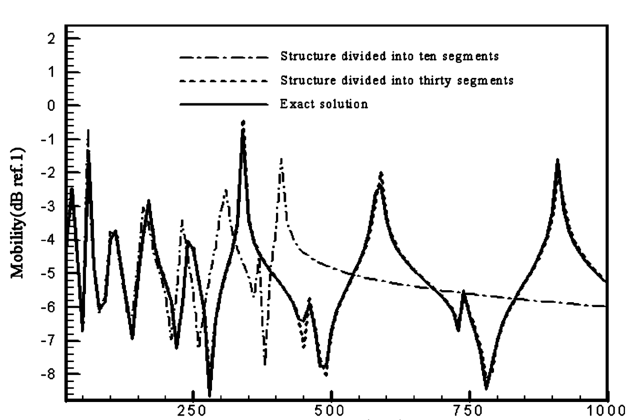

Fig. 3 Driving point mobility of beam at the position of 0.045 m.

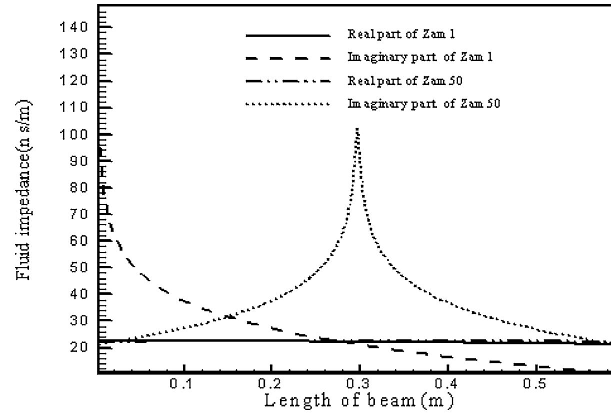

Fig. 4 Acoustic impedance on beam surface at 200 Hz.

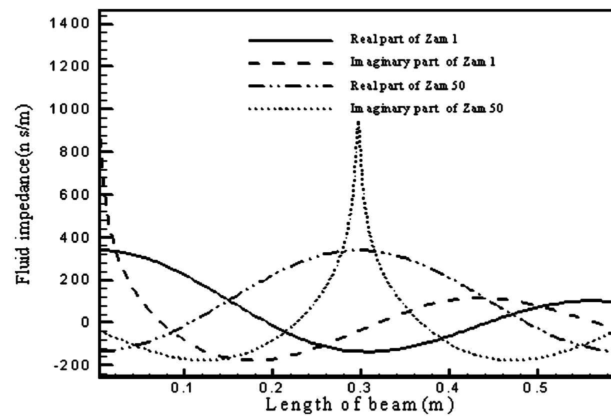

Fig. 5 Acoustic impedance on beam surface at 3000 Hz.

VI. Conclusions

An impedance technique of modeling fluid-loaded structures is proposed. The fluid loading effect is expressed discretely using the frequency response function, and then a formulation that assembles the acoustic impedance and the structural impedance is developed to determine the response of the fluid-loaded structure. The advantage is that the response of the fluid-loaded structure can be found without deriving the equation of motion, which usually encounters difficulties when the structure is either in an irregular shape or with complicated boundary conditions. The fluid loading calculated using the impedance technique is validated numerically, and its accuracy depends on the size of the structural segment relative to the acoustic wavelength. Although this modeling technique is effective provided that the acoustic wavelength is larger than that of the structural segment, however, for a heavy fluid loading the validness of this model can be extended to higher frequencies.

Acknowledgment

This work was partially supported by the National Science Council of Taiwan, Republic of China, under Contract NSC 91-2212-E-194-020.

References

[1]Maidanik, G., "The Influence of Fluid Loading on the Radiation from Orthotropic Plate," *Journal of Sound and Vibration*, Vol. 3, No. 3, 1966, pp. 288–299.

[2]Nayak, P. R., “Line Admittance of Infinite Isotropic Fluid-Loaded Plates,” *Journal of the Acoustical Society of America*, Vol. 78, No. 2, 1970, pp. 763–766.

[3]Geers, T., and Felippa, C. A., “Doubly Asymptotic Approximations for Vibration Analysis of Submerged Structures,” *Journal of the Acoustical Society of America*, Vol. 73, No. 4, 1983, pp. 1152–1159.

[4]Feit, D., and Liu, Y. N., “The Nearfield Response of a Line-Driven Fluid-Loaded Plate,” *Journal of the Acoustical Society of America*, Vol. 47, No. 1, 1985, pp. 191–201.

[5]Crighton, D. G., “Fluid Loading: The Interaction Between Sound and Vibration,” *Journal of Sound and Vibration*, Vol. 133, No. 1, 1989, pp. 1–27.

[6]Junger, M. G., and Feit, D., *Sound, Structures, and Their Interaction*, 2nd ed., MIT Press, Cambridge, MA, 1986, pp. 113–142.

[7]Lee, D., Sternberg, R. L., and Schultz, M. H. (eds.), “Computational Acoustics: Algorithms and Applications,” *Proceedings of the 1st IMACS Symposium on Computational Acoustics*, Yale Univ., 1988.

[8]Richard, F. K., “Structural Acoustic Response of Finite-Reinforced Plates,” *Journal of the Acoustical Society of America*, Vol. 94, No. 2, 1993, pp. 880–887.

[9]Bishop, R. E. D., and Johnson, D. C., *The Mechanics of Vibration*, Cambridge Univ. Press, Cambridge, England, U.K., 1979, pp. 1–52.

[10]Cheng, C. C., and Wang, P. W., “Applications of the Impedance Method on Multiple Piezoelectric Actuators Driven Structures,” *Journal of Vibration and Acoustics*, Vol. 123, No. 2, 2001, pp. 262–268.

[11]Cheng, C. C., and Lin, C. C., “An Impedance Approach for Vibration Response Synthesis Using Multiple PZT Actuators,” *Sensors and Actuators A: Physical*, Vol. 118, No. 1, 2004, pp. 116–126.

[12]Ewins, D. J., *Modal Testing Theory, Practice and Application*, 2nd ed., Research Studies Press, Baldock, England, U.K., 2000, pp. 163–276.

A. Palazotto
Associate Editor